Demystifying problem solving

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Logic Mentoring Workshop

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In seeking "indicators of suspicion" in investigations, suspects will need to have had:

- Motive to commit the crime (for example, financial gain or to seek revenge)
- Means to commit the crime (including tools and physical capabilities)
- Opportunity to commit the crime (including being at the crime scene at the time of the offence)

— Wikipedia on criminal investigation

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Image credit: Sandra Kiefer [Cluedo: German version]

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- No factorisation exists for 5
- `for` d `in` $[2; n)$`:` `if` d `divides` n `then return "`$d \cdot \frac{n}{d}$`"`

Reinventing the wheel/bike

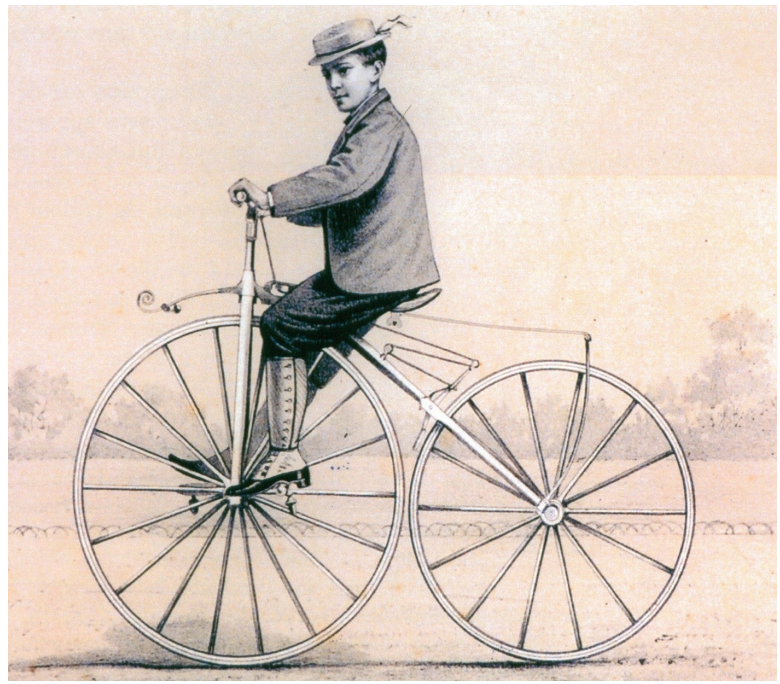

Image source: Wikipedia

Don't get intimidated

George Dantzig (1914–2005)

Image source: mathshistory.st-andrews.ac.uk

Find the right place

Image credit: BBC / IMDB (left); Iona, DO'Neil, Snowmanradio, Andreas Praefcke (all at Wikipedia) (right)

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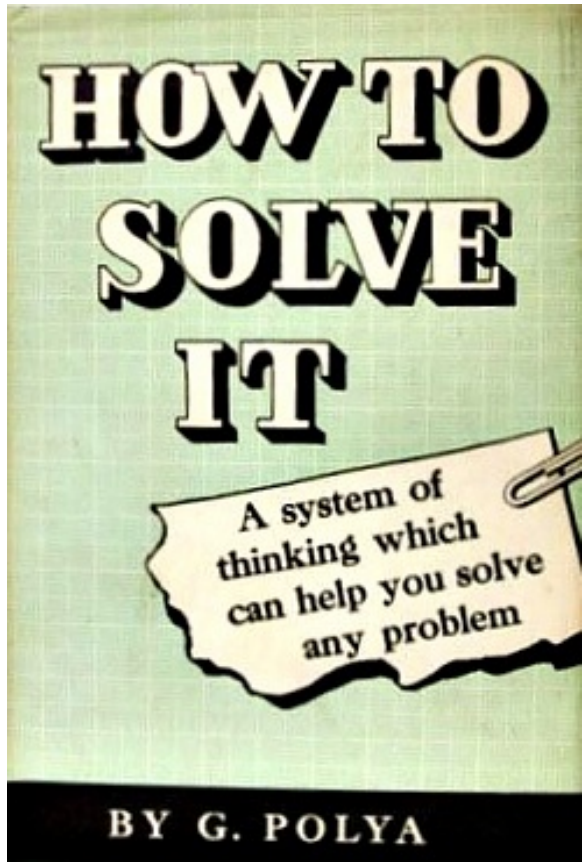

"There is an easier problem that you cannot solve: find it"

Image credit: Princeton University Press (from Wikipedia)

Engage with the problem!
Guess and check // Trial and error

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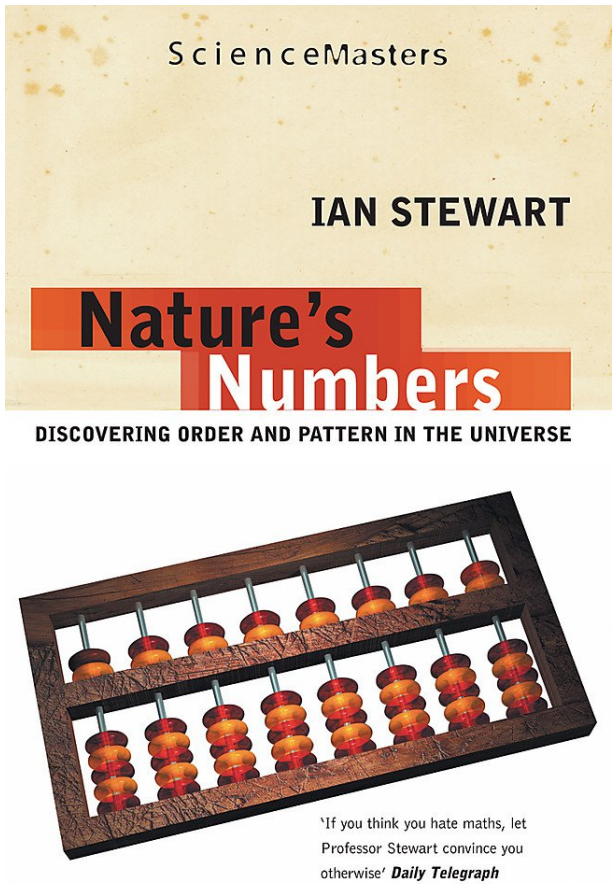
Example: What does the following program do?

for $i := 1$ **to** n: **do**
  for $j := 1$ **to** n: **do**
    if $A[i] < A[j]$: **then**
      swap $A[i]$ and $A[j]$

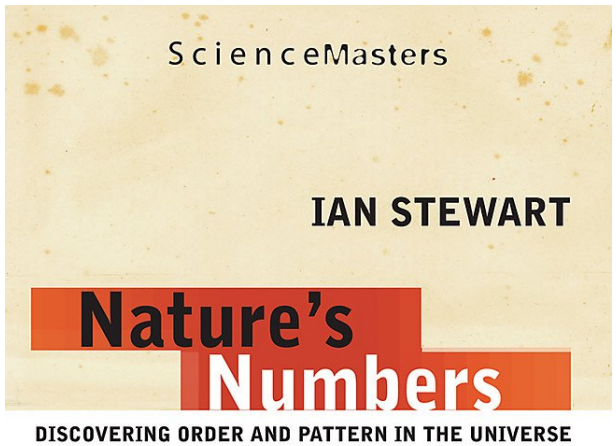
Tactics and strategy

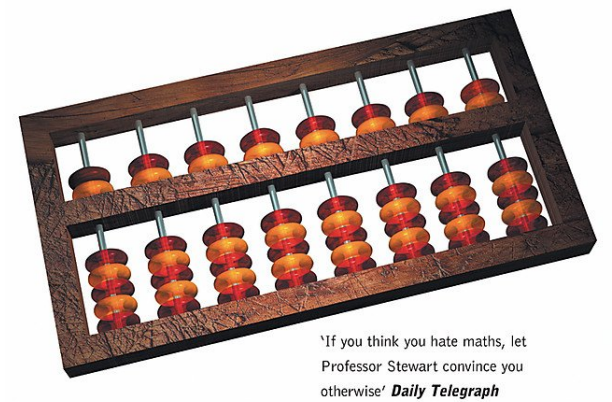


C A T
C O T
C O G
D O G

Image credit: Weidenfeld & Nicolson (via Amazon)

Tactics and strategy





S H I P

C A T
C O T
C O G
D O G

D O C K

Image credit: Weidenfeld & Nicolson (via Amazon)

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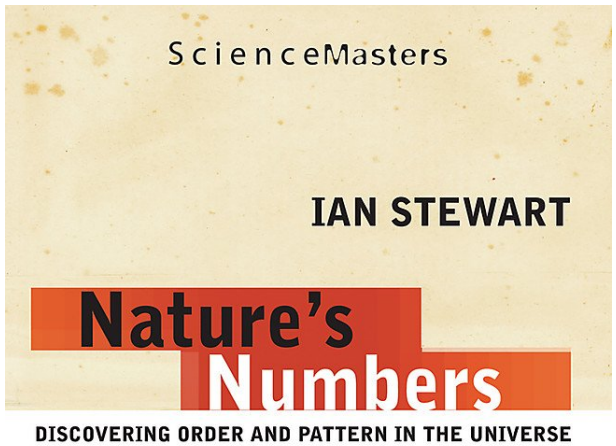

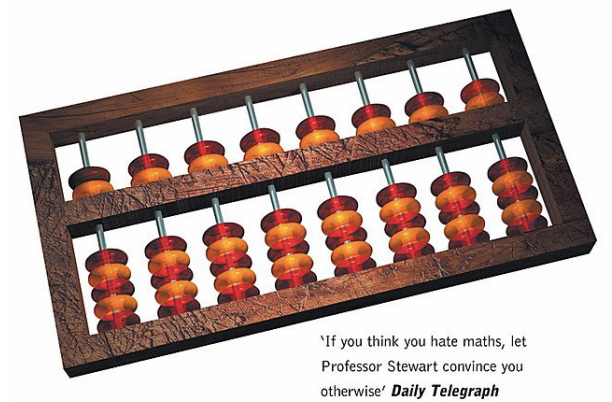

S H I P

C A T
C O T
C O G
D O G

D O C K

Find the right questions!

"How many vowels do intermediate words have?"

Image credit: Weidenfeld & Nicolson (via Amazon)

Tricks of the trade:

- What if the vectors are linearly independent?
- What if we just have an AND of constraints?
- What if we already know what happens with $n - 1$?
- *. . . you name it!*

Image credit: Ron Maijen (Wikipedia), CC BY-SA 3.0

Gobliins 2: The Prince Buffoon
by Coktel Vision

Image source: Tomas Petersen at mobygames.com

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Take time to think!

Image source: Tomas Petersen at mobygames.com

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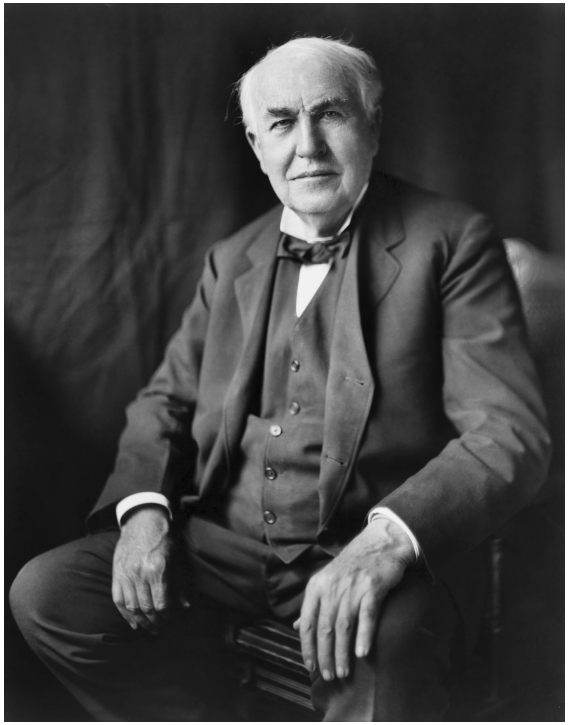
Image credit: Sandra Kiefer [Cluedo: German version]

Mistakes and setbacks

Henri Lebesgue (1875–1941)

Nikolai Luzin (1883–1950)

Image source: Wikipedia



"I have not failed. I have just found 10,000 ways that won't work."

— Attributed to Thomas Edison

"It is impossible to live without failing at something, unless you live so cautiously that you might as well not have lived at all — in which case, you fail by default."

— J. K. Rowling

Image source: Wikipedia — public domain (left), Sjhill, CC BY-SA 3.0 (right)

Impostor syndrome

"I'm a cheat"
"Everyone knows more than I"
"I'm not capable"
"I am only here because of luck"

Impostor syndrome

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Everyone has it.
No, really, everyone does!

Impostor syndrome

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Everyone has it.
No, really, everyone does!

Refute this. Talk to your mentor, supervisor, friends.
You are not alone!

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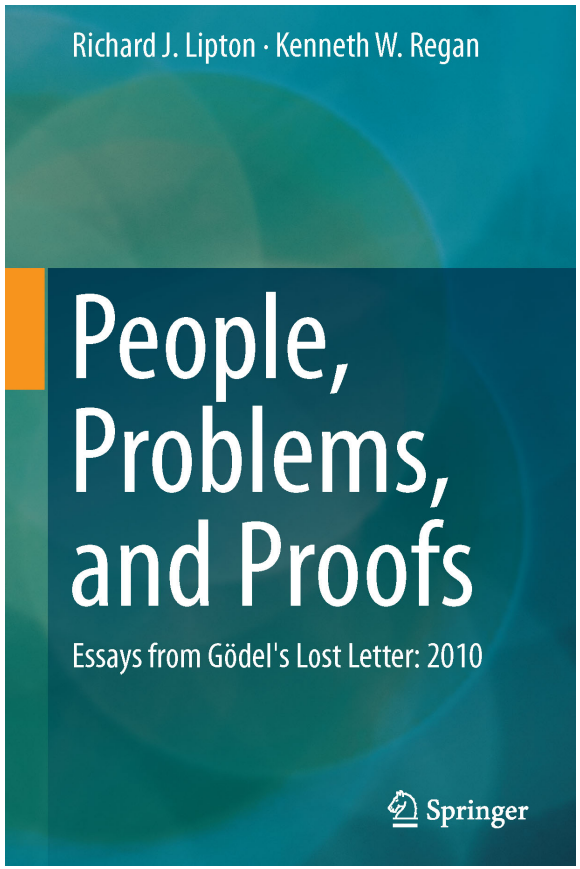
"Every time you talk to a friend about a result, give a talk, or post a paper—let alone publish one—you are sharing your ideas. Most would argue that this is the key to advancing science in all areas."



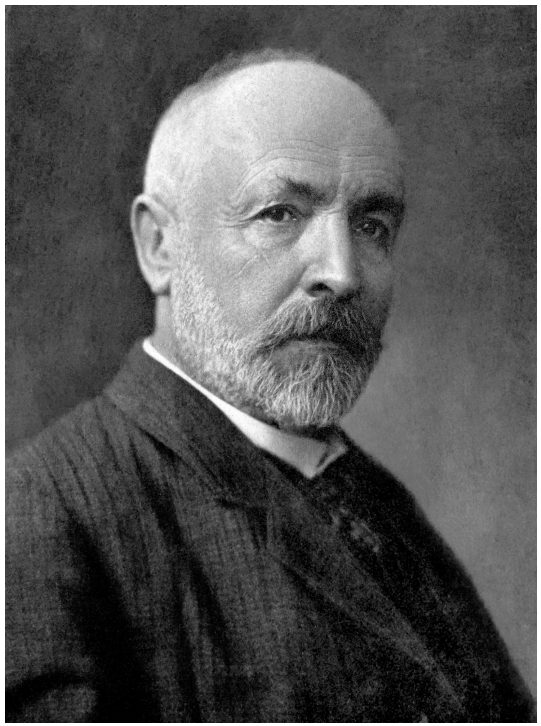
The problem might be difficult inherently.
Fermat's Last Theorem: c. 1637–1995

Pierre de Fermat

Andrew J. Wiles

Image credit: Rolland Lefebvre (public domain), Klaus Barner (CC BY-SA 3.0), both via Wikipedia

What looks simple in retrospect is not always easy to find.



Georg Cantor

1874: $[0,1] \cong [0,1]^2$?

1877:

$$\alpha = 0.a_1a_2 \ldots a_n \ldots$$
$$\beta = 0.b_1b_2 \ldots b_n \ldots$$
$$(\alpha, \beta) \leftrightarrow 0.a_1b_1a_2b_2 \ldots a_nb_n \ldots$$

Things take time.

Image source: Wikipedia

How to solve a problem?

- Understand the problem.
- Identify what works for you.
- Find an easier problem that you still cannot solve.
- Recognise that things take time.
- Remember that you are not alone!

References

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  And: Gödel's lost letter and P=NP. Blog. `https://rjlipton.wordpress.com`.
- Randall Munroe. (Lucky) Ten Thousand. `https://xkcd.com/1053/`
- [On 4-line program, spoilers upfront.]
  `https://news.ycombinator.com/item?id=28758106`.

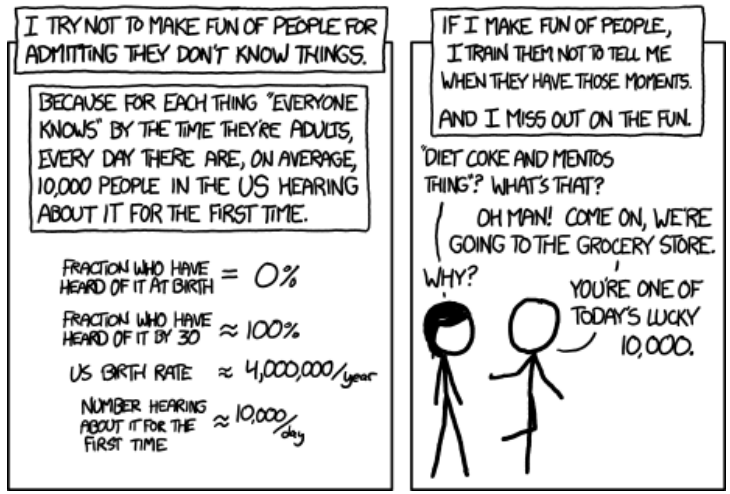

Image credit: xkcd.com